**Exam Cheat Sheet**

**Binomial Coefficient:** For $r \leq n$,

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

represents the number of possible combinations of $n$ objects taken $m$ at a time.

**Multinomial Coefficient:** For $n_1 + n_2 + \cdots + n_r = n$,

$$\binom{n}{n_1, n_2, \ldots, n_r} = \frac{n!}{n_1!n_2! \cdots n_r!}$$

represents the number of possible ways $n$ objects can be divided into $r$ groups with $n_1, n_2, \ldots, n_r$ objects in each, respectively.

**Axioms for Probability Measures:** The following 3 axioms define a probability measure:

1. For all events $E$, $0 \leq P(E) \leq 1$.
2. If $S$ is the sample space, $P(S) = 1$.
3. For any sequence of mutually exclusive events $E_1, E_2, \ldots, E_n$, $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$.

**Useful Formulas for Probability Measures:**

- $P(E^c) = 1 - P(E)$
- $P(E) \leq P(F)$ if $E \subseteq F$
- $P(E) = P(EF)$ if $E \subseteq F$
- $P(E \cup F) = P(E) + P(F) - P(EF)$
- $P(E) = P(EF) + P(EF^c)$
- $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(EF) - P(EG) - P(FG) + P(EFG)$

**Conditional Probability:** The probability of event $E$, given that we observe $F$, is:

$$P(E \mid F) = \frac{P(EF)}{P(F)}$$

Also, $P(E \mid F)$ defines a new probability measure with $F$ being the restricted sample space.

**Independence:** Two items are independent if $P(EF) = P(E)P(F)$. If $P(E \mid F) = P(E)$, $E$ and $F$ are independent; when $P(F) \neq 0$, the two definitions are exactly equivalent.

**Useful Formulas for Contitional Probability:**

- $P(EF) = P(E \mid F)P(F) = P(F \mid E)P(E)$
- $P(E) = P(E \mid F)P(F) + P(E \mid F^c)P(F^c)$
- $P(E_1E_2 \cdots E_n) = P(E_1)P(E_2 \mid E_1)P(E_3 \mid E_2E_1) \cdots P(E_n \mid E_{n-1}E_{n-2} \cdots E_1)$

$$= P(E_n)P(E_{n-1} \mid E_n)P(E_{n-2} \mid E_{n-1}E_n) \cdots P(E_1 \mid E_2E_3 \cdots E_n)$$

$$P(E \mid F) = \frac{P(F \mid E)P(E)}{P(F)}$$