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CORRECTION TO
A GLIVENKO-CANTELLI THEOREM AND STRONG LAWS OF LARGE NUMBERS FOR FUNCTIONS OF ORDER STATISTICS

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Professors Peter Gaenssler and Winfried Stute have kindly brought to my attention an error in the proof of (B) of Theorem 1 of the above-mentioned paper (*Ann. Statist.* **5** (1977) 473–480). The proof given there is valid under the additional assumption that h is concave or convex; or if $h \geqq aI$ for some $a > 0$. It is easily seen however that there exist nonnegative, nondecreasing continuous functions h which satisfy

$$\liminf_{t\to 0} h(t)/t = 0 \qquad \text{and} \qquad \limsup_{t\to 0} h(t)/t = +\infty .$$

These functions are not concave or convex or bounded below by any line through the origin; hence the argument given in the first seven lines of the proof of Theorem 1 is invalid.

Fortunately, part (B) of Theorem 1 is true as stated (without an additional convexity or concavity or boundedness assumption as discussed above). Furthermore, (C) if $\int_0^1 (1/h)\, dI = \infty$ then

$$\limsup_{n\to\infty} \rho_h(\Gamma_n, I) = +\infty \quad \text{w.p. } 1 .$$

The following simple proof of both (B) of Theorem 1 and (C) is due to Gaenssler and Stute.

Without loss suppose $\int_0^\varepsilon (1/h)\, dI = \infty$. Then for any positive integer r and $n \geqq N = N(r, \varepsilon)$ the sequence

$$c_n \equiv \sup\{t \leqq \varepsilon : h(t)^{-1} = 2nr\}$$

is well defined and

$$\sum_{n=n_0}^{\infty} c_n = (2r)^{-1} \sum_{n=n_0}^{\infty} c_n(2(n+1)r - 2nr)$$
$$\geqq (2r)^{-1} \int_0^\varepsilon (1/h)\, dI - \text{constant} = \infty .$$

Thus Borel–Cantelli implies that $P(\xi_n \leqq c_n \text{ i.o.}) = 1$; and consequently $P(\xi_{n1} \leqq c_n \text{ i.o.}) = 1$ also. Since $1/h$ is continuous and nonincreasing on $(0, \varepsilon)$, this implies that

$$\rho_h(\Gamma_n, 0) = \sup_{0<t\leqq 1} (\Gamma_n(t)/h(t)) \geqq (nh(\xi_{n1}))^{-1} \geqq (nh(c_n))^{-1} = 2r$$

infinitely often w.p. 1, which yields (B) of Theorem 1 as claimed. Similarly,

$$\rho_h(\Gamma_n, I) \geqq (2nh(\xi_{n1}))^{-1} \geqq (2nh(c_n))^{-1} = r$$

infinitely often w.p. 1, which proves (C).

It should be noted that (C) together with (A) of Theorem 1 of the paper imply that finiteness of $\int_0^1 (1/h)\, dI$ is both necessary and sufficient for the weighted Glivenko–Cantelli theorem for Γ_n.