Design of Engineering Experiments

The Blocking Principle

- Montgomery text Reference, Chapter 4
- **Blocking** and **nuisance factors**
- The randomized complete block design or the **RCBD**
- Extension of the ANOVA to the RCBD
- Other blocking scenarios…Latin square designs
The Blocking Principle

- **Blocking** is a technique for dealing with *nuisance factors*
- A *nuisance* factor is a factor that probably has some effect on the response, but it’s of no interest to the experimenter…however, the variability it transmits to the response needs to be minimized
- Typical nuisance factors include batches of raw material, operators, pieces of test equipment, time (shifts, days, etc.), different experimental units
- Many industrial experiments involve blocking (or should)
- Failure to block is a common flaw in designing an experiment (consequences?)
The Blocking Principle

- If the nuisance variable is known and controllable, we use blocking.
- If the nuisance factor is known and uncontrollable, sometimes we can use the analysis of covariance (see Chapter 15) to remove the effect of the nuisance factor from the analysis.
- If the nuisance factor is unknown and uncontrollable (a “lurking” variable), we hope that randomization balances out its impact across the experiment.
- Sometimes several sources of variability are combined in a block, so the block becomes an aggregate variable.
The Hardness Testing Example

• Text reference, pg 120
• We wish to determine whether 4 different tips produce different (mean) hardness reading on a Rockwell hardness tester
• Gauge & measurement systems capability studies are frequent areas for applying DOX
• Assignment of the tips to an experimental unit; that is, a test coupon; tip is pressed into a metal test "coupon"; depth of depression yields hardness
• Structure of a completely randomized experiment
• The test coupons are a source of nuisance variability
• Alternatively, the experimenter may want to test the tips across coupons of various hardness levels
• The need for blocking
The Hardness Testing Example

- To conduct this experiment as a RCBD, assign all 4 tips to each coupon
- Each coupon is called a “block”; that is, it’s a more homogenous experimental unit on which to test the tips
- Variability between blocks can be large, variability within a block should be relatively small
- In general, a block is a specific level of the nuisance factor
- A complete replicate of the basic experiment is conducted in each block
- A block represents a restriction on randomization
- All runs within a block are randomized
The Hardness Testing Example

• Suppose that we use \( b = 4 \) blocks:

<table>
<thead>
<tr>
<th>Test Coupon (Block)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Tip 3</td>
</tr>
<tr>
<td>Tip 1</td>
</tr>
<tr>
<td>Tip 4</td>
</tr>
<tr>
<td>Tip 2</td>
</tr>
</tbody>
</table>

• Notice the **two-way structure** of the experiment
• Once again, we are interested in testing the equality of treatment means, but now we have to remove the variability associated with the nuisance factor (the blocks)
Extension of the ANOVA to the RCBD

• Suppose that there are \( a \) treatments (factor levels) and \( b \) blocks

• A **statistical model** (effects model) for the RCBD is

\[
y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \quad \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, b \end{cases}
\]

• The relevant (fixed effects) hypotheses are

\[
H_0 : \mu_1 = \mu_2 = \cdots = \mu_a \quad \text{where} \quad \mu_i = \left(1/b\right)\sum_{j=1}^{b} (\mu + \tau_i + \beta_j) = \mu + \tau_i
\]
Extension of the ANOVA to the RCBD

ANOVA partitioning of total variability:

\[
\sum_{i=1}^{a} \sum_{j=1}^{b} (y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{a} \sum_{j=1}^{b} [(\bar{y}_{i.} - \bar{y}_{..}) + (\bar{y}_{.j} - \bar{y}_{..})] \\
+ (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 \\
= b \sum_{i=1}^{a} (\bar{y}_{i.} - \bar{y}_{..})^2 + a \sum_{j=1}^{b} (\bar{y}_{.j} - \bar{y}_{..})^2 \\
+ \sum_{i=1}^{a} \sum_{j=1}^{b} (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 \\
SS_T = SS_{Treatments} + SS_{Blocks} + SS_E
\]
Extension of the ANOVA to the RCBD

The degrees of freedom for the sums of squares in

\[ SS_T = SS_{Treatments} + SS_{Blocks} + SS_E \]

are as follows:

\[ ab - 1 = a - 1 + b - 1 + (a - 1)(b - 1) \]

Therefore, ratios of sums of squares to their degrees of freedom result in mean squares and the ratio of the mean square for treatments to the error mean square is an \( F \) statistic that can be used to test the hypothesis of equal treatment means.
# ANOVA Display for the RCBD

## Table 4-2 Analysis of Variance for a Randomized Complete Block Design

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>$F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>$SS_{\text{Treatments}}$</td>
<td>$a - 1$</td>
<td>$\frac{SS_{\text{Treatments}}}{a - 1}$</td>
<td>$\frac{MS_{\text{Treatments}}}{MS_{E}}$</td>
</tr>
<tr>
<td>Blocks</td>
<td>$SS_{\text{Blocks}}$</td>
<td>$b - 1$</td>
<td>$\frac{SS_{\text{Blocks}}}{b - 1}$</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>$SS_E$</td>
<td>$(a - 1)(b - 1)$</td>
<td>$\frac{SS_E}{(a - 1)(b - 1)}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$SS_T$</td>
<td>$N - 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Vascular Graft Example (pg. 124)

- To conduct this experiment as a RCBD, assign all 4 pressures to each of the 6 batches of resin.
- Each batch of resin is called a “block”; that is, it’s a more homogenous experimental unit on which to test the extrusion pressures.

<table>
<thead>
<tr>
<th>Extrusion Pressure (PSI)</th>
<th>Batch of Resin (Block)</th>
<th>Treatment Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8500</td>
<td>90.3</td>
<td>89.2</td>
</tr>
<tr>
<td>8700</td>
<td>92.5</td>
<td>89.5</td>
</tr>
<tr>
<td>8900</td>
<td>85.5</td>
<td>90.8</td>
</tr>
<tr>
<td>9100</td>
<td>82.5</td>
<td>89.5</td>
</tr>
<tr>
<td>Block Totals</td>
<td>350.8</td>
<td>359.0</td>
</tr>
</tbody>
</table>
# Vascular Graft Example

“Design-Expert” Output

## Response: Yield

### ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>DF</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Prob &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Block</td>
<td>192.25</td>
<td>5</td>
<td>38.45</td>
<td></td>
<td>0.0019</td>
</tr>
<tr>
<td>Model</td>
<td>178.17</td>
<td>3</td>
<td>59.39</td>
<td>8.11</td>
<td>0.0019</td>
</tr>
<tr>
<td>A</td>
<td>178.17</td>
<td>3</td>
<td>59.39</td>
<td>8.11</td>
<td>0.0019</td>
</tr>
<tr>
<td>Residual</td>
<td>109.89</td>
<td>15</td>
<td>7.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cor Total</td>
<td>480.31</td>
<td>23</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Std. Dev.: 2.71
- Mean: 89.80
- C.V.: 3.01
- PRESS: 281.31

- R-Squared: 0.6185
- Adj R-Squared: 0.5422
- Pred R-Squared: 0.0234
- Adeq Precision: 9.759
Residual Analysis for the Vascular Graft Example

Figure 4-4  Normal probability plot of residuals for Example 4-1.
Residual Analysis for the Vascular Graft Example

Figure 4-5 Plot of residuals versus $y$ for Example 4-1.

Figure 4-6 Plot of residuals by extrusion pressure (treatment) and by batches of resin (block) for Example 4-1.
Residual Analysis for the Vascular Graft Example

• Basic residual plots indicate that normality, constant variance assumptions are satisfied
• No obvious problems with randomization
• No patterns in the residuals vs. block
• Can also plot residuals versus the pressure (residuals by factor)
• These plots provide more information about the constant variance assumption, possible outliers
Multiple Comparisons for the Vascular Graft Example – Which Pressure is Different?

| Treatment | Estimated Mean  | Standard Error | t for H₀ | Prob > |t||
|-----------|----------------|----------------|----------|----------|----------|
| 1 vs 2    | 1.13           | 1.56           | 0.73     | 0.4795   |
| 1 vs 3    | 3.90           | 1.56           | 2.50     | 0.0247   |
| 1 vs 4    | 7.05           | 1.56           | 4.51     | 0.0004   |
| 2 vs 3    | 2.77           | 1.56           | 1.77     | 0.0970   |
| 2 vs 4    | 5.92           | 1.56           | 3.79     | 0.0018   |
| 3 vs 4    | 3.15           | 1.56           | 2.02     | 0.0621   |

Also see Figure 4-3, Pg. 128
Other Aspects of the RCBD
See Text, Section 4-1.3, pg. 130

• The RCBD utilizes an additive model – no interaction between treatments and blocks
• Treatments and/or blocks as random effects
• Missing values
• What are the consequences of not blocking if we should have?
• **Sample sizing** in the RCBD? The OC curve approach can be used to determine the number of blocks to run..see page 131