1) $E \subset F^c$ means that $E$ is outside $F$ and thus $E \cap F = \emptyset$. Thus $P(E \cap F^c) = 1 - P(E \cup F) = 1 - P(E) - P(F) \Rightarrow a$.

2) $E^c \subset F$ implies that $F$ covers the outside of $E$, $E \cup F = S \Rightarrow d$.

3) The number of distinct words is
$$\binom{7}{1, 2, 1, 1, 2} = \frac{7!}{1!2!1!1!2!} = 1260 \Rightarrow d.$$ 

4) Since $A = AE \cup AE^c$ and $AE \cap AE^c = \emptyset$, we have $P(A) = P(AE) + P(AE^c), \Rightarrow c$.

5) $P(AB) = P(A) + P(B) - P(A \cup B)$ is just a scramble of $P(A \cup B) = P(A) + P(B) - P(AB), \Rightarrow b$.

6) a) Let $D_i$ denote the event that I roll die $i$. Then
$$P\left(\{(1, 2, 3)\} | D_3\right) = \frac{2}{6} \cdot \frac{1}{3} = \frac{1}{9},$$ and
$$P\left(\{(1, 2, 3)\} | D_1\right) = \frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18},$$ hence
$$P\left(\{(1, 2, 3)\}\right) = \frac{1}{3} P\left(\{(1, 2, 3)\} | D_1\right) + \frac{1}{3} P\left(\{(1, 2, 3)\} | D_2\right) + \frac{1}{3} P\left(\{(1, 2, 3)\} | D_3\right)$$
$$= \frac{1}{6} \left(\frac{1}{3} + \frac{1}{3} + \frac{2}{3}\right) = \frac{1}{6} \cdot \frac{4}{3} = \frac{2}{9}.$$

b) $P(D_3 | \{(1, 2, 3)\}) = \frac{P\left(\{(1, 2, 3)\} | D_3\right) P(D_3)}{P\left(\{(1, 2, 3)\}\right)} = \frac{(2/6)^3(1/3)}{(1/6)^3(4/3)} = \frac{1}{2}$.

7) Let $A$ be the event of getting the queen of spades or the ace of hearts or both. Then $A^c$ is the event of getting a hand without those two cards, i.e., $P(A^c) = \binom{52}{13} / \binom{52}{13}$ and $P(A) = 1 - P(A^c) = .441176$.

8 a) Let $A$ denote the event that the seal is missing and write $E = F_1^c F_2^c F_3$ for short. Then
$$P(E | A) = .3^2 \cdot .7 = .063, \quad \text{and} \quad P(E | A^c) = .999^2 \cdot .001 = .000998.$$

b) $P(A | E) = \frac{P(E | A) P(A)}{P(E | A) P(A) + P(E | A^c) P(A^c)} = \frac{.063 \cdot .02}{.063 \cdot .02 + .000998 \cdot .98} = .563$. 

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