Space-time growth-interaction processes motivated by tree growth

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People involved

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- Claudia Redenbach, University of Kaiserslautern, Germany
- Jun Yu, Umeå University, Sweden
- Kenneth Nyström, Swedish University of Agricultural Sciences, Umeå, Sweden
- Henrike Häbel, Chalmers and University of Gothenburg, Sweden
Outline

- Growth-interaction process
- Estimation of parameters
- Scots pine data
- Future work
Growth-interaction process

- Marked point processes evolving in time
- New immigrants (trees) arrive according to a Poisson process, with rate $\alpha$, have uniformly distributed locations, and are assigned initial marks (sizes)
- In small time intervals $(t, t + dt)$, each individual either dies ‘naturally’ with probability $\mu dt$, or undergoes the deterministic size change

$$m_i(t + dt) = m_i(t) + f(m_i(t))dt$$
$$+ \sum_{j \neq i} h(m_i(t), m_j(t), \|x_i - x_j\|)dt,$$

where $f(\cdot)$ individual growth function, $h(\cdot)$ spatial interaction function, and $\|x_i - x_j\|$ distance between trees $i$ and $j$

- Remark: Interactive death if the mark (size) becomes negative
Growth functions

- **Linear** growth function
  
  \[ f(m_i(t)) = \lambda(1 - m_i(t)/K), \]

  where \( \lambda \) is the intrinsic growth rate and \( K \) the carrying capacity

- **Logistic** growth function
  
  \[ f(m_i(t)) = \lambda m_i(t)(1 - m_i(t)/K) \]

- Both the **linear** and the **logistic** growth functions are special cases of the so-called **logistic power-law** function
  
  \[ f(m_i(t)) = c_1 m_i(t) - c_2 (m_i(t))^{p+1} \]
Interaction functions

- **Symmetric interaction function**

  \[
  h(m_i(t), m_j(t), \|x_i - x_j\|) = -bl(\|x_i - x_j\| < r(m_i(t) + m_j(t)),
  \]
  
  where \( b > 0 \) is the strength of interaction, \( r \) is the scale of interaction, and \( I(x) \) denotes the indicator function (see e.g. Renshaw and Särkkä, 2001).

- **Non-symmetric area-interaction function**

  \[
  h(m_i(t), m_j(t), \|x_i - x_j\|) = -b \frac{|D(x_i, rm_i(t)) \cap D(x_j, rm_j(t))|}{\pi r^2 m_i^2(t)},
  \]
  
  where \( |\cdot| \) denotes area and \( D(x, m) \) a disk with radius \( m \) centered at \( x \) (see Gerrard, 1969; Särkkä and Renshaw, 2006).

- Both models are useful in forestry
Remarks

- Instead of uniform arrivals the new immigrants can arrive according to a hard-core process (SSI) or due to seeding.
- Initial marks can be taken e.g. from $U(0, \epsilon)$, $\epsilon > 0$ (small), or they can be fixed.
- The model is very flexible, almost anything can be changed.
The sample area is located in a developing woodland near Stockholm (more plots available).

Data: tree locations and diameters at breast height (dbh) recorded in a circular plot of radius 10m in 1985, 1990 and 1996. All trees greater than 10cm in diameter are included.

Exact arrival (and death) times of the trees are unknown.
Scots pine data

Pines 1985

Radii 1985

Pines 1990

Radii 1990

Pines 1996

Radii 1996
ML estimators for arrival rate \( \alpha \) and death rate \( \mu \) (without taking into account the exact way of collecting the data; estimates biased downwards)

Least squares (LS) estimates for the growth and interaction parameters \( \lambda, K, r \) and \( b \) by minimizing (w.r.t. the parameters)

\[
S_M = \sum_{t=2}^{T} \sum_{i \in \Omega_t} (\tilde{m}_i(t) - m_i(t))^2,
\]

where \( \Omega_t \) denotes the tree pattern at time \( t \), \( \tilde{m}_i \) is the size based on the model, and \( m_i \) the observed size
Edge correction added to the LS procedure: given the initial estimates (obtained without edge correction), data simulated in an area outside the sample plot taking the observed data into account.

We also gave a new estimator for $\alpha$ by taking into account that the points arriving and dying during the same time interval are not observed.
Improved LS estimation (with Claudia Redenbach)

- Spatial structure of the point (tree) locations included in the estimation procedure
- Minimize
  \[ S = w_1 S_1 + w_2 S_2 + \ldots + w_k S_k, \]
  where
  - \( S_1, S_2, \ldots, S_k \) are functions depending on marks, locations or on both
  - \( w_j, j = 1, \ldots, k \) are non-negative weights such that \( \sum_{j=1}^{k} w_j = 1 \)

The estimates for the growth and interaction parameters can be found by minimizing \( S \) with respect to the parameters.
Distance measure

We minimize $S = 0.5(S_M + S_L)$, where

$$S_M = \frac{1}{T-1} \sum_{t=2}^{T} \frac{1}{|\Omega_t|} \sum_{i \in \Omega_t} \left( \frac{\tilde{m}_i(t) - m_i(t)}{\max(m_i)} \right)^2,$$

where $\max(m_i)$ is the largest observed mark, and

$$S_L = \frac{1}{(T-1)M} \sum_{t=2}^{T} \sum_{i=1}^{M} \left( \frac{\tilde{L}_t(s_i) - L_t(s_i)}{L_t(s_i)} \right)^2,$$

where $\tilde{L}_t$ and $L_t$ are the $L$ functions of the simulated model and of the data, respectively, in the $t$-th observed time step, and the $L$ functions are evaluated in $M$ distances $s_1, \ldots, s_M$. 
How does the approach work for the Scots pine data?

- Logistic power-low growth function, area-interaction function and hard-core arrivals
- Parameters estimated with the improved LS method with edge correction
- Estimated model: logistic growth with growth rate $\lambda = 0.10$ and carrying capacity $K = 0.095$, strength of interaction $b = 0$ and range of interaction $r = 10.1$
Spatial locations and size histograms

Data 1996

Model 1996

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Spatial summary statistics

Top: pair correlation function
Bottom: mark correlation function
Conclusions

- The spatial summary statistics (pair correlation and mark correlation function) look fine
- Model based mark distribution is very different from the observed one
Estimation of open growth

- Ottmar Cronie, Kenneth Nyström, and Jun Yu: Estimated the carrying capacity $K$ separately from an open growth data (as a linear function of so-called site index), and plugged in the estimate when estimating the remaining parameters.

- Henrike Häbel: Estimated the whole growth function (Richards growth function) separately from an open growth data by fitting a non-linear mixed model.

- Neither of the approaches give satisfying results concerning the growth-interaction process.
What to do next?

- Size distribution?
- Include data from 2004 (and more plots)
- LS estimation and distance measure
- ML estimation
- Applications from materials science