# Part V
# Chance Variability

## Chapter 16
## The Law of Averages

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## Overview

We will look at various chance processes:

- Tossing coins, rolling dice, playing roulette
- Sampling voters

We will use something called box models to analyze these processes. Box models help to translate real life problems into statistical problems.

The questions we'll answer in Chapters 16-18 are of the following type: Suppose we play a game of roulette 10 times.

- What is our expected net gain?
- How much variability do we expect?
- What is the chance that we will come out ahead?
- How do these answers depend on the number of times we play?

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## Coin tossing experiment

A coin lands heads or tails with equal chances of 50%. In the long run, should the number of heads equal the number of tails?

John Kerrich, a South African mathematician, tried this out in practise. He was visiting Copenhagen when WWII broke out. The Germans invaded Denmark and he wasn't allowed to leave. Instead, he spent the war in an internment camp in Jutland where he had plenty of time for experiments. One thing he did was to toss a coin 10,000 times and count the number of heads.

The results are given in Section 16.1 in the textbook.

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## Coin tossing experiment

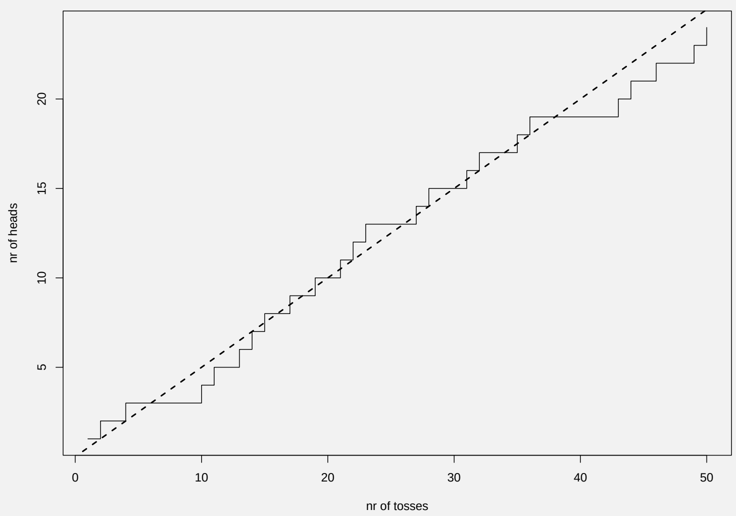
It is not very likely that John Kerrich got exactly 5000 heads. We expect that he got about 5,000 heads.

Number of heads = half the number of tosses + chance error

As the number of tosses goes up, the chance error gets bigger in absolute terms.

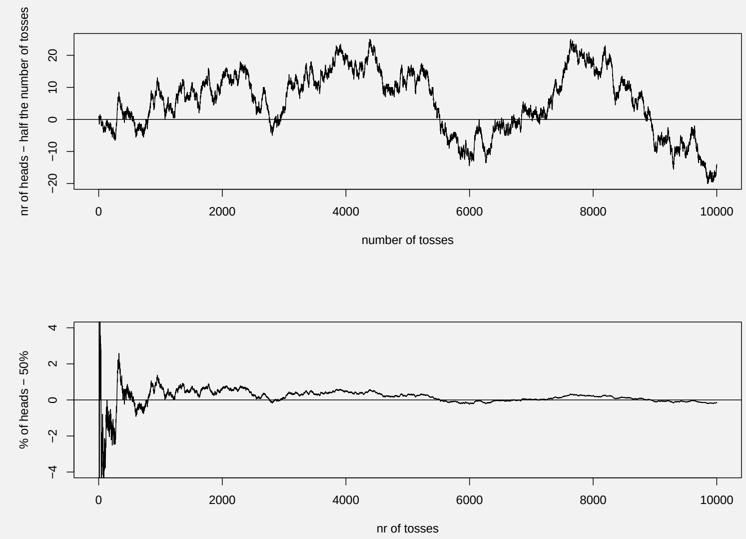
However, if we express the chance error as a percentage of the number of tosses, it gets smaller.

## First 50 tosses






## 10,000 tosses






## 10,000 tosses

| nr tosses | observed nr of heads | chance error | observed % of heads | chance error in % |
|---|---|---|---|---|
| 50 | 24 | -1 | 48.00% | −2.00% |
| 100 | 47 | -3 | 47.00% | −3.00% |
| 500 | 252 | 2 | 50.40% | 0.40% |
| 1000 | 508 | 8 | 50.80% | 0.80% |
| 5000 | 2514 | 14 | 50.28% | 0.28% |
| 10000 | 4986 | -14 | 49.86% | −0.14% |



## Law of averages

### Definition

As the number of tosses goes up, the difference between the number of heads and half the number of tosses gets bigger; but the differnce between the percentage of heads and 50% gets smaller.

This is called the law of averages.

*Note*: The law of averages does not work by changing the chances. After a long run of heads, a head is still as likely as a tail in the next toss.

## Chance processes

Looking at the number of heads in a coin-tossing experiment is an example of a problem about chance processes.

Other examples are:

1. the amount of money lost or won at roulette
(the chance process is spinning the wheel)
2. the percentage of Democrats in a random sample of voters
(the chance process is used to draw the sample)



## Chance processes

Our general strategy to analyze this sort of processes relies on two main ideas:

1. find an analogy between the process begin studied (sampling voters in the poll example) and drawing numbers at random from a box
2. connect the variability you want to know about (e.g. in the estimate for the Democratic vote) with the chance variability in the sum of the numbers drawn from the box



## The box model

### Definition

The analogy between the chance process and drawing from a box is called a box model.

*Why a box model?* Because the chance variability in the sum of numbers drawn from a box can be analyzed easily.

It helps to translate a real life process into a statistical problem. The box model contains only the relevant information, and we strip away all the irrelevant stuff.



## Example 1: rolling two dice

Consider the box

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |

Draw two tickets with replacement, and compute the sum of the two draws

- What is the lowest possible value?
- What is the highest possible value?
- What is a likely value?

What is this a box model for? For the number of squares you move in a game of Monopoly (roll a pair of dice, and count the total number of spots).

## The sum of the draws

This kind of problem will be solved soon, and 'the sum of the draws' is a shorthand for the process discussed here:

- draw tickets at random with replacement from a box
- add up the numbers on the tickets

*Examples:*

1. The number of squares you'll move in a turn at Monopoly is like the sum of two draws from a box with tickets 1,2,3,4,5,6.
2. The number of heads in 10,000 coin tosses is like the sum of 10,000 draws from a box with tickets 0,1.

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## Making a box model

There are three questions to answer when making a box model:

1. What tickets go in the box?
2. How many of each ticket?
3. How many draws do we make?

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## Example 2: American roulette

An American roulette wheel has 38 pockets

- one is numbered 0 and another 00 (green)
- the rest is numbered from 1 to 36 (alternating red and black)

Players make various types of bets. Croupier spins wheel, and throws ball on wheel. The ball is equally likely to land in any of the 38 pockets.

If you bet $1 on red, say, and a red number comes up, you get the dollar back with another dollar in winning. Otherwise, the croupier takes your dollar.

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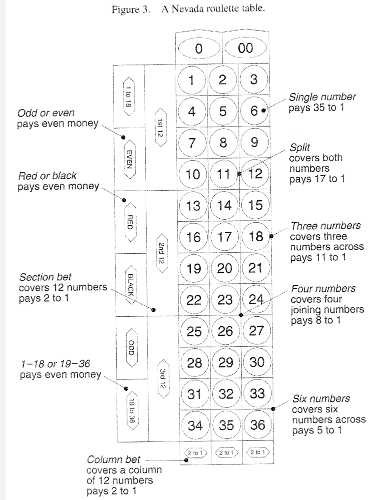
## Example 2: American roulette

Figure 3. A Nevada roulette table.




*Roulette is a pleasant, relaxed, and highly comfortable way to lose your money.*
—JIMMY THE GREEK

## Example 2: American roulette

The box model for betting $1 on red:

| 18 tickets | + $1 | 20 tickets | - $1 |
|---|---|---|---|

1. What tickets go in the box?
   *The tickets in the box show the various amounts that can be won or lost on a single play*
2. How many of each ticket?
   *The chance of drawing any particular number must equal the chance of winning that amount on a single play ('Winning' a negative number means losing)*
3. How many draws do we make?
   *The number of draws equals the number of plays*

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## Example 2: American roulette

| Play | Amount won in each play | Net gain |
|---|---|---|
| red | $1 | $1 |
| red | $1 | $2 |
| red | $1 | $3 |
| black | −$1 | $2 |
| green | −$1 | $1 |
| red | $1 | $2 |
| red | $1 | $3 |
| black | −$1 | $2 |
| black | −$1 | $1 |
| red | $1 | $2 |

Net gain after these 10 plays: $2.

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## Summary of the box model for gambling problems

- Tickets in the box show the amounts that can be won (+) or lost (-)
- The chance of drawing any particular value from the box equals the chance of winning that amount in a single play
- The number of draws equals the number of plays
- The net gain is the sum of the draws from the box

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## Summary for today's lecture

Today, we have learned about

- the chance error.
- how the error is likely to be large in absolute terms, but small realtive to the number of plays. That is the law of averages.
- the box model: descriping chance processes with 'drawing tickets from a box'.
- the three basic questions to ask when making a box model.