1.0 Random Variables and Distributions

1.1 Counting Rules

- Factorial Notation: If n is a positive integer, define $n!$ ('n factorial') by

$$n! = n(n-1)(n-2) \ldots (2)(1)$$

Define $0! = 1$.

Ex. $3! = 3 \cdot 2 \cdot 1 = 6$.

- **Counting Rule I**

How many ways can I draw x items from a list of n items if the draws are made sequentially and the drawn item is is not replaced after each draw? In comparing the sets the order of selection is important.

- **Counting Rule II (permutation rule)**

How many ways can I permute n items? That is, how many distinct orderings of n items exist.

- **Counting Rule III**

How many ways can I draw x items from a list of n items if the draws are made sequentially and the drawn item is is replaced after each draw?

- **Counting Rule:** The number of different ways of choosing x objects from a total of n objects is

$$\binom{n}{x} = \frac{n!}{x!(n-x)!} = \frac{n(n-1)\ ...\ (n-x+1)}{x!}$$

- $\binom{n}{x}$ ('n choose x') is the number of subsets of size x which can be formed from a set of n objects.

Ex. Consider the set of $n = 3$ issues that people think are important, $\{Education\ ,\ Drugs\ ,\ Crime\}$. Suppose we must designate the two most important to us (without regard to relative importance). There are three possible sets of two issues from this set, namely $\{Education\ ,\ Drugs\}$, $\{Education\ ,\ Crime\}$, $\{Drugs\ ,\ Crime\}$. This result is consistent with the above counting rule, since $\binom{3}{2} = 3$.

Ex. What is the probability of obtaining a flush (5 cards of the same suit) in a 5 card poker hand dealt from a fresh deck?

1.2 The expected value of a random variable