

# Lecture 2

## *Exponents*

Based on notes by Barbara E. Crist  
West Virginia University

The rules of positive integer exponents introduced in the basic algebra course are revisited in subsequent algebra courses as negative integer, real number, and rational exponents are introduced. The rules for exponent operations with several examples are as follows:

Rule - multiplication:  $a^m a^n = a^{m+n}$

Examples:  $x^3 x^4 = x^7$   
 $2^4 \cdot 2^{-1} = 2^3$   
 $a^{2/3} a^{1/4} = a^{11/12}$  (Note: Remember to find the LCD)

Rule:  $a^0 = 1$   $a \neq 0$

Examples:  $(-2)^0 = 1$   
 $(4x^2 y)^0 = 1$

Rule – negative exponents:  $a^{-n} = \frac{1}{a^n}$   $a \neq 0$

Examples:  $2^{-3} = \frac{1}{2^3}$   
 $x^2 y^{-5} = x^2 \cdot \frac{1}{y^5} = \frac{x^2}{y^5}$   
 $2x^{-4} = \frac{2}{x^4}$  (Note: The negative exponent is only for x.

The exponent on the number 2 is a positive one.)

Rules- powers:  $(a^m)^n = a^{mn}$  and  $(a \cdot b)^n = a^n b^n$

Examples:  $(x^{-3})^4 = x^{-12} = \frac{1}{x^{12}}$

$$(4x^2y^{-3})^{-3} = 4^{-3}x^{-6}y^9 = \frac{y^9}{64x^6}$$

$$\left(x^{2/3}\right)^{6/5} = x^{4/5}$$

Rules – division:  $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$  ,  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$  and  $\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$

Examples:  $\frac{4^5}{4^2} = 4^3$  (Note: The base does not change when the division is performed.)

$$\frac{x^{1/3}}{x^{1/4}} = x^{1/3-1/4} = x^{1/12}$$

$$\left(\frac{4x^{-2}y^4}{x^5y^{-2}}\right)^{-3} = \left(\frac{x^5y^{-2}}{4x^{-2}y^4}\right)^3 = \frac{x^{15}y^{-6}}{64x^{-6}y^{12}} = \frac{x^{21}}{64y^{18}}$$

The rules for exponents can be applied in a different order in the previous example. The problem could have been worked as follows with the following result.

$$\left(\frac{4x^{-2}y^4}{x^5y^{-2}}\right)^{-3} = \frac{4^{-3}x^6y^{-12}}{x^{-15}y^6} = \frac{x^{21}}{64y^{18}}$$

Mistakes made working exponent problems are usually sign mistakes or mistakes in the arithmetic used with fractional exponents. Remember that any type of problem with exponents follows the rules from the basic algebra class.