Lecture 3
Radicals

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$\sqrt[n]{a} = b$ which is equivalent to $b^n = a$ is a radical with index n. These radicals can be simplified by taking the nth root of a if possible or factoring a into nth powers if possible.

Please note: Because the principal square root is always used, always make sure that your answer to an even root is positive by using the absolute value for any variable that may not be positive. So $\sqrt{a^2} = |a|$.

Examples: $\sqrt[3]{64} = 4$

$\sqrt{16} = 4$ (Note: A missing index is assumed to be 2 or square root.)

Simplifying Radicals

Expressions containing radicals must be simplified. The quantity under the radical is factored into perfect squares, cubes, or into powers equal to the index of the radical. Any quantity that can be removed from under the radical is then removed.

$$\sqrt{8} = \sqrt{2 \cdot 4} = 2\sqrt{2}$$

Examples: $\sqrt[3]{81} = \sqrt[3]{27 \cdot 3} = 3\sqrt[3]{3}$

$$\sqrt{32x^4 y^7} = \sqrt{16 \cdot 2 \cdot x^4 \cdot y^6 \cdot y} = 4x^2 y^3 \sqrt{2y}$$

Multiplication Property
$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

The multiplication property may be used in the multiplication and simplification of radicals.

$$\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$$

Examples: $\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

$$\sqrt{2}\left(\sqrt{3} + \sqrt{6}\right) = \sqrt{6} + \sqrt{12} = \sqrt{6} + \sqrt{4 \cdot 3} = \sqrt{6} + 2\sqrt{3}$$

Quotient Property

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Examples:

$$\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$$

$$\frac{\sqrt{xy^3}}{\sqrt{xy}} = \sqrt{\frac{xy^3}{xy}} = \sqrt{y^2} = |y|$$

> The quotient property may be used to separate numerator and denominator into separate radicals or to make two radicals one.

Division of Radicals – Rationalizing the denominator

When radicals appear in the denominator, rationalize the denominator by multiply the top and bottom of the fraction by whatever is necessary to make the denominator a perfect square or cube.

Examples:

$$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{\sqrt{6}}{3}$$

$$\frac{\sqrt{7}}{\sqrt{8}} = \frac{\sqrt{7}}{\sqrt{2 \cdot 4}} = \frac{\sqrt{7}}{2\sqrt{2}} = \frac{\sqrt{7}\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{\sqrt{14}}{2\sqrt{4}} = \frac{\sqrt{14}}{4}$$

$$\frac{\sqrt{3}}{\sqrt{5} - \sqrt{3}} = \frac{\sqrt{3}\left(\sqrt{5} + \sqrt{3}\right)}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})} = \frac{\sqrt{15} + \sqrt{9}}{\sqrt{25} - \sqrt{9}} = \frac{\sqrt{15} + 3}{5 - 3} = \frac{\sqrt{15} + 3}{2}$$

Addition and Subtraction with radicals

When adding and subtracting expressions with radicals, like terms are combined. Like radical terms have the same radicals and the same variable coefficients.

$$2\sqrt{x} + 3\sqrt{x} = 5\sqrt{x}$$

Examples: $5xy\sqrt{xy} - 3x\sqrt{xy} + 4x\sqrt{xy} = 5xy\sqrt{xy} + x\sqrt{xy}$