CSSS 505

Calculus Summary Formulas

Differentiation Formulas

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(fg) = fg' + gf'$
3. $\frac{d}{dx}(\frac{f}{g}) = \frac{gf' - fg'}{g^2}$
4. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$
5. $\frac{d}{dx}(\sin x) = \cos x$
6. $\frac{d}{dx}(\cos x) = -\sin x$
7. $\frac{d}{dx}(\tan x) = \sec^2 x$
8. $\frac{d}{dx}(\cot x) = -\csc^2 x$
9. $\frac{d}{dx}(\sec x) = \sec x \tan x$
10. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
11. $\frac{d}{dx}(e^x) = e^x$
12. $\frac{d}{dx}(a^x) = a^x \ln a$
13. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
14. $\frac{d}{dx}(Arc\sin x) = \frac{1}{\sqrt{1-x^2}}$
15. $\frac{d}{dx}(Arc\tan x) = \frac{1}{1+x^2}$
16. $\frac{d}{dx}(Arc\sec x) = \frac{1}{|x|\sqrt{x^2-1}}$
17. $\frac{dy}{dx} = \frac{dy}{dx} \times \frac{du}{dx}$ *Chain Rule*

Trigonometric Formulas

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$
4. $\sin(-\theta) = -\sin\theta$
5. $\cos(-\theta) = \cos\theta$
6. $\tan(-\theta) = -\tan\theta$
7. $\sin(A+B) = \sin A\cos B + \sin B\cos A$
8. $\sin(A-B) = \sin A\cos B - \sin B\cos A$
9. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
10. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
11. $\sin 2\theta = 2\sin\theta\cos\theta$
12. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
13. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cot\theta}$
14. $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$
15. $\sec\theta = \dfrac{1}{\cos\theta}$
16. $\csc\theta = \dfrac{1}{\sin\theta}$
17. $\cos(\dfrac{\pi}{2} - \theta) = \sin\theta$
18. $\sin(\dfrac{\pi}{2} - \theta) = \cos\theta$

Integration Formulas

Definition of a Improper Integral

$\int_a^b f(x)\,dx$ is an improper integral if

1. f becomes infinite at one or more points of the interval of integration, or
2. one or both of the limits of integration is infinite, or
3. both (1) and (2) hold.

1. $\int a\,dx = ax + C$
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{x}\,dx = \ln|x| + C$
4. $\int e^x\,dx = e^x + C$
5. $\int a^x\,dx = \frac{a^x}{\ln a} + C$
6. $\int \ln x\,dx = x\ln x - x + C$
7. $\int \sin x\,dx = -\cos x + C$
8. $\int \cos x\,dx = \sin x + C$
9. $\int \tan x\,dx = \ln|\sec x| + C$ or $-\ln|\cos x| + C$
10. $\int \cot x\,dx = \ln|\sin x| + C$
11. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
12. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$
13. $\int \sec^2 x\,dx = \tan x + C$
14. $\int \sec x \tan x\,dx = \sec x + C$
15. $\int \csc^2 x\,dx = -\cot x + C$
16. $\int \csc x \cot x\,dx = -\csc x + C$
17. $\int \tan^2 x\,dx = \tan x - x + C$
18. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} Arc\tan\left(\frac{x}{a}\right) + C$
19. $\int \frac{dx}{\sqrt{a^2 - x^2}} = Arc\sin\left(\frac{x}{a}\right) + C$
20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} Arc\sec\frac{|x|}{a} + C = \frac{1}{a} Arc\cos\left|\frac{a}{x}\right| + C$

Formulas and Theorems

1a. Definition of Limit: Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. Then $\lim_{x \to a} f(x) = L$ means that for each $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

1b. A function $y = f(x)$ is continuous at $x = a$ if

i). $f(a)$ exists

ii). $\lim_{x \to a} f(x)$ exists

iii). $\lim_{x \to a} = f(a)$

4. Intermediate-Value Theorem

A function $y = f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ and $f(b)$.

Note: If f is continuous on $[a,b]$ and $f(a)$ and $f(b)$ differ in sign, then the equation $f(x) = 0$ has at least one solution in the open interval (a,b).

5. Limits of Rational Functions as $x \to \pm\infty$

i). $\lim_{x \to \pm\infty} \dfrac{f(x)}{g(x)} = 0$ if the degree of $f(x) <$ the degree of $g(x)$

Example: $\lim_{x \to \infty} \dfrac{x^2 - 2x}{x^3 + 3} = 0$

ii). $\lim_{x \to \pm\infty} \dfrac{f(x)}{g(x)}$ is infinite if the degrees of $f(x) >$ the degree of $g(x)$

Example: $\lim_{x \to \infty} \dfrac{x^3 + 2x}{x^2 - 8} = \infty$

iii). $\lim_{x \to \pm\infty} \dfrac{f(x)}{g(x)}$ is finite if the degree of $f(x) =$ the degree of $g(x)$

Example: $\lim_{x \to \infty} \dfrac{2x^2 - 3x + 2}{10x - 5x^2} = -\dfrac{2}{5}$

6. Average and Instantaneous Rate of Change

i). *Average Rate of Change*: If (x_0, y_0) and (x_1, y_1) are points on the graph of $y = f(x)$, then the average rate of change of y with respect to x over the interval $[x_0, x_1]$ is $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0} = \dfrac{y_1 - y_0}{x_1 - x_0} = \dfrac{\Delta y}{\Delta x}$.

ii). *Instantaneous Rate of Change*: If (x_0, y_0) is a point on the graph of $y = f(x)$, then the instantaneous rate of change of y with respect to x at x_0 is $f'(x_0)$.

7. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

8. <u>The Number ***e*** as a limit</u>

i). $$\lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^{n} = e$$

ii). $$\lim_{n \to 0} \left(1 + \frac{n}{1}\right)^{\frac{1}{n}} = e$$

9. <u>Rolle's Theorem</u>

If f is continuous on $[a,b]$ and differentiable on (a,b) such that $f(a) = f(b)$, then there is at least one number c in the open interval (a,b) such that $f'(c) = 0$.

10. <u>Mean Value Theorem</u>

If f is continuous on $[a,b]$ and differentiable on (a,b), then there is at least one number c in (a,b) such that $\dfrac{f(b) - f(a)}{b - a} = f'(c)$.

11. <u>Extreme-Value Theorem</u>

If f is continuous on a closed interval $[a,b]$, then $f(x)$ has both a maximum and minimum on $[a,b]$.

12. To find the maximum and minimum values of a function $y = f(x)$, locate

1. the points where $f'(x)$ is zero <u>or</u> where $f'(x)$ fails to exist.
2. the end points, if any, on the domain of $f(x)$.

<u>Note</u>: These are the <u>only</u> candidates for the value of x where $f(x)$ may have a maximum or a minimum.

13. Let f be differentiable for $a < x < b$ and continuous for a $a \le x \le b$,

1. If $f'(x) > 0$ for every x in (a,b), then f is increasing on $[a,b]$.
2. If $f'(x) < 0$ for every x in (a,b), then f is decreasing on $[a,b]$.