HW1: Combinatorial Analysis

Directions. Show and explain all work to receive full credit. Homework is due on Friday, October 7th at the beginning of class.

Problem 1. In some school, the 500 students have to learn at least one foreign language among the following ones: Arabic, French, German, Japanese, Spanish and Swedish. According to the pigeonhole principle, we know that at least 2 students will learn the same language. Using a corollary seen in class, what would be a correct claim on the number of students of a given language class?

Problem 2. A person picks at random one pair of jeans, one t-shirt and one pair of socks. Assume that she owns 5 pairs of jeans, 2 of which are black, 6 t-shirts, 4 of which are black, 8 pairs of socks, 5 of which are black.

(a) In how many ways can that person dress up?
(b) In how many ways can she dress up entirely in black?

Problem 3. On a given afternoon, a group of friends plan to either watch a movie in the theater or visit a museum. But they cannot watch a movie and visit a museum in the same afternoon. In the town, there are 6 movies playing in the theater and 2 museums. How many different options do they have for the afternoon? Justify your answer.

Problem 4.

(a) Give an analytical proof of the following property: for any \( n, p \in \mathbb{N} \), with \( p \leq n \),

\[
\binom{n+1}{p+1} = \binom{n}{p} + \binom{n}{p+1}.
\] (1)

(b) Provide a combinatorial argument (i.e., explain with “words”) for the following property: for any \( n, p \in \mathbb{N} \), with \( p \leq n \),

\[
\binom{n}{p} = \binom{n}{n-p}.
\] (2)
**Problem 5.** A soccer team is composed of 11 players.

(a) Assume that each of the 11 players is assigned to only one position on the field of play. How many different configurations of team can the coach make up?

(b) Actually, there are 4 main categories of positions: goalkeeper, defensive, midfield, and attacking positions. Assume that the coach wants 1 goalkeeper, 4 defensive players, 3 midfield players and 3 attacking players. In how many ways can the coach assign each of the 11 players to one of the 4 categories?

(c) After extra time of a soccer game, the coach has to select 5 penalty kickers among the 11 players. In how many ways can the coach pick the 5 kickers?

(d) Actually, the coach has to decide which order the kicks are taken. How many different ordered lists of 5 penalty kickers are there?

**Problem 6.** A student association has to elect among its members a committee consisting of one president, one vice-president and one secretary. The association is composed of 8 girls and 5 boys.

(a) How many different committees are possible?

*Hint. Consider the two following outcomes:*

- Mark, president / Julia, vice-president / Christine, secretary;
- Christine, president / Julia, vice-president / Mark, secretary.

*Those correspond to two different committees although they include the same people.*

How many different committees are possible if

(b) the president and the vice-president must be of opposite sexes?

(c) the secretary must be a boy?

(d) both sexes must be represented in the committee?